



## IDENTIFICATION OF APPROXIMATE, GENERALISED VARIABLES FOR FORMULATION OF FIELD DATA BASED MODEL OF PVC MANUFACTURING PROCESS: THEROTICAL APPROACH

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### Abstract

The paper describes an approach for identification of approximate, generalized independent and dependent variables for formulation of field data based mathematical model (FDMM) for the process of PVC pipe manufacturing of an enterprise. The aim of field data based modeling for PVC pipe manufacturing process is to improve the performance of system by correcting or modifying the inputs for improving output. The work identifies major approximate, generalized process parameters and other workstation related parameters which will affect the production output and quality of the PVC pipe extrusion process using single screw extruder. Some of the identified parameters are raw materials, formation of PVC compound, in-barrel moisture, screw speed, barrel temperature, extrusion die characteristics, screw design, viscosity, torque, bulk density, die pressure, power, screw diameter, screw length, screw pitch, L/D ratio etc. Out of all the variables identified, dependent and independent variables of the PVC pipe manufacturing system are determined. A mathematical relationship is established between dependent variables as an output parameters and independent variables as a input parameters. The mathematical relationship exhibit in which input variables is to be maximized or minimized to optimize output variables. Once model is formulated it can be optimized using the optimization technique further. Sensitivity analysis is a tool which can be used to find out the effect of input variables on output variables. Simultaneously it would be interesting to know influence of one parameter over the other. The model will be useful for an entrepreneur of an industry to select optimized inputs so as to get targeted responses.

### Concept of field databased mathematical model

Field data based modeling is applicable for any type of man-machine system [1].Field data based model form the relationship between input and output variables. This type of modeling is used for improving the performance of system by suggesting or modifying the inputs for improving output. The mathematical model formed is useful in selecting optimized input variables so as to reduce some of the unwanted input independent variables to improve the production output and quality [10].

### Introduction to PVC pipes Extrusion

The extruder can be considered as one of the core piece of machinery in the polymer processing industry. To extrude means to thrust out a polymer material in any kind of form with a desired cross section through a die. The shape of material will depend on the die opening, and it will change to some extent as it exits from the die. The extruded output is commonly referred to as the extrudate [3]. Plastic pipes are presented in different forms, they could be corrugated, water pipes, electricity pipes, high pressure pipes for many demanding applications and more pipes with special requirements. Pipe and profile production line consists (figure1) of an extruder which is equipped with a die depending on the end product and also a calibration device [8]. After the extruder, the material is run through a cooling pool, nip rolls and a cutting saw. Biggest production of pipes is made out of PVC, PE and PP. During the manufacturing process of pipes, the molten plastic from extruder is led to a circle die towards the calibration device, where the final shape and size are determined. Pipe calibration can be carried out either in high or low pressure. Nip roll's function is to pull the calibrated product towards the cooling pool, which is then cut into desired length [8].

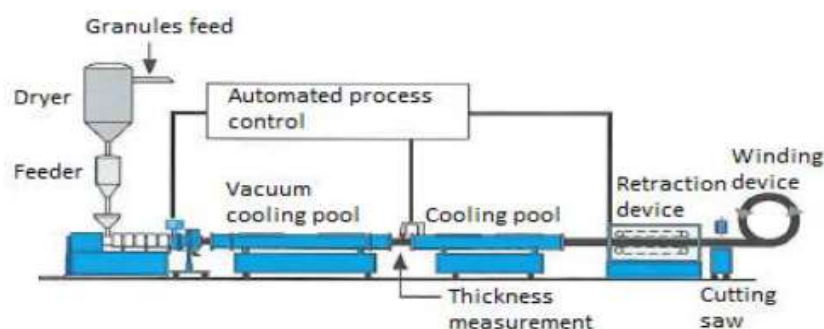




Figure: 1 PVC pipe manufacturing

**Study of process of PVC pipe manufacturing on Single Screw Extruders**

Most extruders are equipped with screw as their main mixing component. Screw extruders are classified as single or multiple screw extruders. Single screw extruders are the most common type of extruders used in the polymer industry, because of its straight forward design, relatively low costing, and its reliability, they are most often used (Mad-dock, [4]). Their screw has only one compression section, even though the screw has three distinct geometrical sections; the first section (closest to the feed opening) generally has deep flights. The material in this section will be mostly in the solid state. This section is referred to as the feed section of the screw. The last section usually has shallow flights; the material in this section will be mostly in the molten state. This screw section is referred to as the metering section. This section is also called the transition section or compression section. In most cases, the depth of the screw channel reduces in a linear fashion, going from the feed section towards the metering section, thus causing a compression of the material in the screw channel [3].

The extruder is a very important machine in the plastic industry, as compared to injection molding machines, they are used to produce product of continuous profile. Extruder's main components are a hopper at one end from which the material to be extruded is fed, a tubular barrel, usually electrically heated; a revolving screw, ram or plunger within the barrel, and a die at the opposite end for shaping the extruded mass [5]. The different parts that makes up an extruder are reviewed below Figure 2.

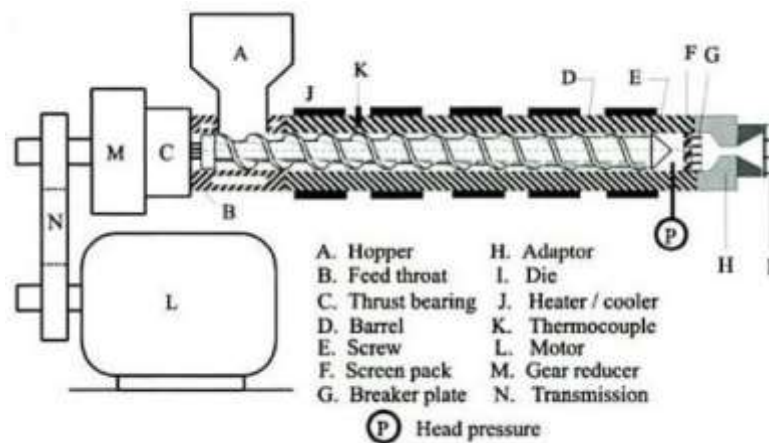


Figure2: Component of a typical single screw extruder

A typical extrusion line (figure 3) consists of the material feed hopper, basic extruder (drive, gearbox and screws), the extrusion die, the calibration units, the haul-off, the saw (or other cutting device), and finally the treatment devices for final finishing and handling [1].

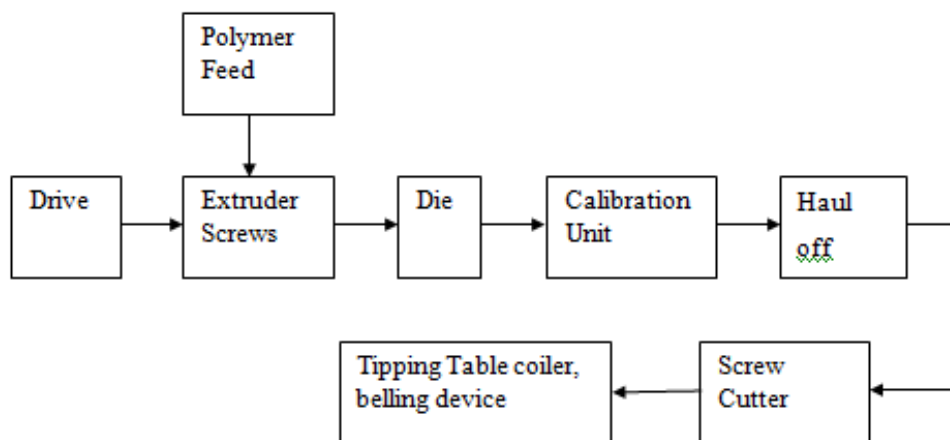


Figure 3: typical extrusion line

The hopper holds the raw plastic material (in either powder or granule form) and continuously feeds this into the extruder, which has a heated barrel containing the rotating screw. This screw transports the polymer to the die head and simultaneously the material



is heated, mixed, pressurized and metered. At the die the polymer takes up the approximate shape of the article and is then cooled either by water or air to give the final shape. As the polymer cools it is drawn along by haul-off devices and either coiled (for soft products) or cut to length (for hard products).

## Study for identification of independent, dependent and extraneous variables

The term variables are used in a very general sense to apply any physical quantity that undergoes change. If a physical quantity can be changed independent of the other quantities, then it is an independent variable. If a physical quantity changes in response to the variation of one or more number of independent variables, then it is termed as dependent or response variable. If a physical quantity that affects our test is changing in random and uncontrolled manner, then it is called an extraneous variable [11]. following are some of the approximate, generalized process variables.

### Raw material

The main raw material required is compounded PVC resin. Presently both PVC & Polyethylene plastics raw materials are available. Other compounding materials like plasticizers, stabilizers, lubricants and fillers are also available. The raw materials required are: (1) PVC resin (2) DOP (3) stabilizer (4) processing acids (5) Colourant (6) filler.

### The L: D ratio

The L: D ratio of an extruder screw is defined as the length of an extruder screw with respect to its diameter. An L: D ratio of 24:1 is common, but some bigger machines go up to 32:1 for faster mixing and additional output with no change in the diameter of the screw [5]. The range of L/D (length to diameter) ratios gives adequate shear, optimum melt homogeneity, and high production rates. Lower L/D ratios give lower outputs.

### The Drive power

Historically the extruder uses the DC motor as the main driving component, but this system had been replaced by an AC motor. This AC drives are digital and brushless. The AC motor can easily be optimized into a 3-phase motor when need arise. AC motors are rated between 25 kW to 85 kW and is controlled by drives with parallel ratings; depending on the extruder model. The rotation of the motor is transferred to the screw with the help of the shaft and pulley system to get maximum transmission efficiency even under adverse high-torque conditions. The performance of a drive system is also very important to the extruder machine and should be thought of when considering the type of machine to purchase [3].

### Die characteristics

The extruder die is the component of the machine that gives the final shape of the product, so its design is crucial both for specification and accuracy. The extruder die create a passage through which molten polymer exit the extruder with the help of pressure built up in the barrel during processing. The molten polymer coming out of the die has a constant exit velocity across the entire die exit.. If the flow of the polymer is not constant it may result to the degradation of the stagnated polymer along the barrel channel after a considerable period of time [3].

### Extruder die pressure

The accurate knowledge of extruder pressure is very important for efficient extrusion. Measuring pressure is important for safety reasons if it gets too high, the head and die might blow off and hurt or damage people or machines nearby. Pressure is good for mixing, especially in the last (metering) zone in single -screw systems. However, higher pressure also means more energy is taken through the motor, thus higher melt temperature, which may dictate the pressure limit.

### Extruder barrel temperature

Extruders require large heater ratings to decrease heating time. A typical barrel zone electrical heater can be rated anywhere from 1.2 to 11 kW and greater [6]. The size generally is determined by the type of feed, the screw type, and the resin feed location. External heating of the barrels can be provided by electric resistance heaters and by hot oil systems. A typical three zone barrel temperature control is adequate for extrusion. Each zone should be equipped with heating and cooling. Digital temperature controllers with thermocouples embedded in the barrel will produce accurate temperature control. It is suggested to place a melt temperature probe at the exit of the screw. These temperature controls will allow the maintenance of the melt temperature of at the desired level. The typical temperature settings for the extrusion are presented in the table 1 below [2]

*Table 1: The typical temperature settings for the extrusion*

Extruder	Setting °C
Feeding zone	135



Compression zone	175-185
Metering zone	195-200
Head	200
Die	210

### Melt temperature

Typical melt temperature between 195 and 220 °C is optimum [2]. A too low melt temperature will result in increased die swell and rough surface because of lack of homogeneity of the melt. A too high melt temperature is detrimental to the dimensional stability and to the drawdown of the extrudate. Melt temperatures above 250 °C should be avoided as they might cause thermal degradation.

### Die temperature

Accurate temperature control of the die is essential to produce high quality extrusions. It is important to have uniform temperatures throughout the adapter and the die body. Dies are typically heated externally with mica or ceramic insulated electric heater bands or cartridges. External bands are the easiest method for heating smaller cylindrical die bodies. Large or more complex dies may require a combination of cartridges and bands with varying watt densities to produce uniform heating. Proportional (PID) temperature controllers should be used with thermocouples for temperature feedback. Simple dies require a single temperature zone. The zone should utilize a thermocouple embedded in the die body close to the melt channel. Wide sheet dies may have several heater zones across the width of the die. A separate temperature controller should be used for the adapter.

### In-barrel moisture

Moisture as a variable works as: plasticizer, viscosity modifier. Impact of moisture increase: Viscosity decrease, torque decrease, product temperature decrease, bulk density increase (expansion) die pressure decrease.

### Polymer viscosity

Polymer viscosity variations of the feed constitute the most difficult aspect of extruder design, simulation, and control. The feed viscosity tends to have more influence on the extrusion process and on the dimensional quality of the pelletizing extruder output than all other variables.

### Screw speed

In most extruders, screw speed is changed by modifying motor speed. Motors typically turn at around 1750 rpm at full speed, but this is much too fast for an extruder screw [7]. If it were turned that fast, it would generate too much frictional heat, and the residence time of the plastic would be too short to prepare a uniform, well-mixed melt. A typical reduction ratio is between 10:1 and 20:1. The first stage may use either gears or a pulley set, but the second stage always uses gears and the screw is set in the center of the last, big gear. High shear rates are recommended in order to obtain low melt viscosities. Screw speed helps control the shear rate. Minimum screw speed: 20 rpm, Typical screw speed: 25 to 80 rpm, depending on the size of the extruder. Up to 200 rpm can be used for most screws without generating excessive melt temperature. However, surface quality of the extrudate may deteriorate as screw speed increases. Larger screws tend to operate at lower speed.

### Screw diameter

It depends on the type of extrudate and on the desired [2]. Small profiles are normally run on smaller machines (25 to 65 mm screw diameter). Linear extrusion speeds of 10 to 30 meters/minute are typical. This speed varies strongly according to the complexity of the extrudate. In order to increase the production capacity, it is possible to carry out duplex or multiple extrusions of simple section profiles. For larger extrudate, it is advisable to use bigger extruders, with screw diameters around 65-114 mm. Smaller or larger screw diameters may also be selected if the intended production requires it. The extruder should be sized appropriately such that a screw speed in the range 25 to 80 rpm is used. If the extrudate cannot be run within this speed range, then a smaller or larger screw diameter should be selected. To obtain best surface appearance of the extrudate, it is recommended to use a bigger extruder with a lower screw speed rather than a small extruder running at high speed.

### Power

Despite the popular fascination and the real problems on a plant level with rising power costs, the power needed to run. This will always be so because material cost is much higher, an extruder is an efficient system and if excess energy is introduced the plastic will soon get too hot to process properly [7].



### Production output

It is also useful to calculate output per rpm, as this shows any deterioration of the screw's pumping capacity with time. Another related calculation is the output per hp or kW of power used. This is the efficiency and enables estimation of the production capacity of a given motor and drive. Output is an important factor in determining the size of the desired extruder for a given application. Output depends on several variables, among others: screw diameter, screw speed, screw configuration, presence of a grooved barrel, type of extrudate/design of profile, die back pressure, temperature profile, and melt viscosity [2].

### List of parameters affecting the output parameters

Some of the approximate, generalized process parameter which affects the production rate and accuracy of PVC pipe manufactured were selected. At preliminary stage, more parameters were selected based upon the material, screw design, raw material properties. Later on to reduce the variables in number, some variables are clubbed together while some variables which was seemed to be uncontrollable or least affecting, that parameter was rejected and finally a set of attributes were selected as shown in following table.

*Table 2: list of independent and dependent variables*

<b>A. Production output related variables</b>				
<b>Sr.No</b>	<b>Description of variables</b>	<b>Type of variable</b>	<b>Symbol</b>	<b>Dimension</b>
1	Production output ( $Y_1$ ) (Quantity of pipe extruded)	Dependent	$Q_o$	M
2	Outer diameter of screw	Independent	$d_o$	L
3	Inner diameter of screw	Independent	$d_i$	L
4	Diameter of barrel	Independent	$d_b$	L
5	Screw RPM	Independent	$W_e$	$T^{-1}$
6	Pitch of the screw	Independent	P	L
7	Input energy to extruder	Independent	E	$M L^{-2} T^{-2}$
8	Quality of raw material	Independent	$Q_R$	M
9	Instantaneous time	Independent	t	T
10	Torque of screw	independent	$T_{shaft}$	$M L^2 T^{-2}$
11	Quantity Raw material	Independent	RM	M
12	Power	Independent	P	$M L^2 T^{-3}$
<b>B. Variables affecting Quality of PVC pipe</b>				
<b>Sr.No</b>	<b>Description of variables</b>	<b>Type of variable</b>	<b>Symbol</b>	<b>Dimension</b>
1.	Thickness of pipe ( $Y_2$ )	Dependent	$T_p$	$M^0 L^1 T^0$
2.	Length of the pipe	independent	S	L
3	L/D ratio	Independent	G	$M^0 L^0 T^0$
4	Hardness of screw	Independent	BHN	$M^0 L^0 T^0$
5	Temperature of barrel	independent	$T_B$	$M^0 L^0 T^0$



6	Temperature of die	independent	T <sub>D</sub>	M <sup>0</sup> L <sup>0</sup> T <sup>0</sup>
7	Viscosity of melt	Independent	μ	M L <sup>-1</sup> T <sup>-1</sup>
8	Density of melt	Independent	ρ	M L <sup>-3</sup>
9	Bulk Modulus	Independent	K	M L T <sup>-2</sup>
10	Pressure in barrel	Independent	p	M L <sup>-1</sup> T <sup>-2</sup>
11	Force (thrust) in die	independent	F	M L <sup>-2</sup>

**Selection of primary dimensions**

According to Theories of engineering experimentation by H. Schenck Jr. “The choice of primary dimensions”, most systems require at least three primaries, but the analyst is free to choose any reasonable set he wishes, the only requirement being that his variables must be expressible in his system [13]. In PVC pipe manufacturing process all the variables are expressed in mass (M), length (L), time (T) hence M, L, and T are chosen as primary dimensions for the dimensional analysis.

**Development of mathematical model by reduction of variables – Dimensional analysis**

Dimensional analysis is the best known and the most powerful technique of reducing the number of variables and making the experimental plan compact without loss of generality or control [12]. Dimensional analysis, basically, helps in deciding algebraic relationship amongst the various physical quantities encountered in the process. Using Buckingham π theorem following dimensional equation is formed.

1. The general equation form of dependent and independent variables are as follows.

$$Y_1 = \Phi_1 [(Q_o)^a, (d_o)^b, (d_i)^c, (d_b)^d, (W_e)^e, (P)^f, (E)^g, (Q_R)^h, (t)^i, (T_{shaft})^j, (RM)^k, (P)^l] \text{ -----(1)}$$

$$Y_2 = \Phi_2 [(T_p)^m, (S)^n, (G)^o, (BHN)^p, (T_B)^q, (T_D)^r, (\mu)^s, (\rho)^t, (K)^u, (p)^v, (F)^w] \text{ -----(2)}$$

2. The MLT form of the above equation is

$$Y_1 = \Phi_1 [(M)^a, (L)^b, (L)^c, (L)^d, (T^{-1})^e, (L)^f, (M L^{-2} T^{-2})^g, (M)^h, (T)^i, (M L^2 T^{-2})^j, (M)^k, (M L^2 T^{-3})^l] \text{ -----(3)}$$

$$Y_2 = \Phi_2 [(L)^m, (M^0 L^0 T^0)^n, (M^0 L^0 T^0)^o, (M^0 L^0 T^0)^p, (M^0 L^0 T^0)^q, (M L^{-1} T^{-1})^r, (M L^{-3})^s, (M L T^{-2})^t, (M L^{-1} T^{-2})^u, (M L^{-2})^v] \text{ -----(4)}$$

The mathematical relation between inputs and outputs could be of any form may be polynomial, exponential or log linear. The Buckingham Pi theorem found suitable for developing the model. As it states that if the inputs and outputs represented in dimensionless pie terms by dimensional analysis then they can be represented as

$$\Pi_5 = \Phi_1 \times (\Pi_1)^a \times (\Pi_2)^b \text{ -----(5)}$$

$$\Pi_6 = \Phi_2 \times (\Pi_3)^c \times (\Pi_4)^d \text{ -----(6)}$$

Where

Π1 and Π2 are pi-terms related with production output related independent variable

Π3 and Π4 are pi-terms related with independent variables affecting the quality of PVC pipe.

Π5 is the dependent variable production output.

Π6 is the dependent variable quality of the PVC pipe.

The model has various indices as Φ1, Φ2, a, b, c, d whose values are unknown. This model represents the field data base model for two workstation for PVC pipe manufacturing process. This model and the Pi terms formed in this model will be used for developing model for different workstation in the process. The Pi terms will be different for other workstations. There are six unknown terms in the equation (5) and (6) curve fitting constant Φ1, Φ2 and indices a, b, c, d, e. To get the values of these unknowns we need minimum a set of five set of all unknown dimensionless pi terms.





**Simplification of the model and determining the indices**

Model contains various indices as  $\Phi_1, \Phi_2, a, b, c, d$  whose values are unknown. The values for dependent and independent Pi terms can be easily find out by observation and recording of workstation parameters [10]. For determining the indices of the relation between output and inputs multiple regressions and Mat lab software can be used. There are six unknown terms in the equation (5) and (6) curve fitting constant  $\Phi_1, \Phi_2$ , and indices  $a, b, c, d$ . To get the values of these unknowns we need minimum a set of five set of all unknown dimensionless pi terms.

Consider the following relation

$$Z = A + bX + cY \text{-----(7)}$$

Equation (7) represents equation of a curve fitting technique. Equation (5) and (6) can be brought in the form of equation (7) as follows.

Taking log on both side of equation (5) and (6)

$$\text{Log } \Pi_5 = \text{log } \Phi_1 \times a \text{ log } (\Pi_1) \times b \text{ log } (\Pi_2) \text{----- (8)}$$

$$\text{Log } \Pi_6 = \text{log } \Phi_2 \times c \text{ log } (\Pi_3) \times d \text{ log } (\Pi_4) \text{----- (9)}$$

Let,  $Z_1 = \text{Log } \Pi_5, K_1 = \text{log } \Phi_1, A = \text{log } (\Pi_1), B = \text{log } (\Pi_2)$  and

$$Z_2 = \text{Log } \Pi_6, K_2 = \text{log } \Phi_2, C = \text{log } (\Pi_3), D = \text{log } (\Pi_4)$$

Equation (8) and (9) becomes

$$Z_1 = K_1 + a \times A + b \times B \text{----- (10)}$$

$$Z_2 = K_2 + c \times C + d \times D \text{----- (11)}$$

Equation (10) and (11) is a regression equation of  $Z_1$  and  $Z_2$  on  $A, B, C, D$  in a dimensional co-ordinate system Again equation (4) is written as

$$Z_1 = n \times K_1 + a \times A + b \times B \text{----- (12)}$$

$$\Sigma Z A = K_1 \Sigma A + a \times \Sigma A \times A + b \times \Sigma B \times A \text{----- (13)}$$

$$\Sigma Z B = K_1 \Sigma B + a \times \Sigma A \times B + b \times \Sigma B \times B \text{----- (14)}$$

and

$$Z_2 = n \times K_2 + c \times C + d \times D \text{----- (15)}$$

$$\Sigma Z C = K_2 \Sigma C + c \times \Sigma A \times C + d \times \Sigma B \times C \text{----- (16)}$$

$$\Sigma Z D = K_2 \Sigma D + c \times \Sigma A \times D + d \times \Sigma B \times D \text{----- (17)}$$

In the above set of equations the values of the multipliers  $K_1, K_2, A, B, C, D$  are substituted to compute the values of  $a, b, c, d$ . After substituting these values in the equations one will get a set of four equations, which are mutinously to get the values of  $K_1, K_2, a, b, c, d$ . The above equations can be verified in the matrix form and further values of  $K_1, K_2, a, b, c, d$  can be obtained by using matrix analysis.

$$[P1] = [W1] [X1] \text{-----(18)}$$

$$X1 = \text{inv } (W1) \times P1 \text{----- (19)}$$

$$[P2] = [W2] [X2] \text{-----(20)}$$

$$X2 = \text{inv } (W2) \times P2 \text{----- (21)}$$

Where,  $W1 = 4 \times 4$  matrix multipliers of  $k1, a, b$  and  $W2 = 4 \times 4$  matrix multipliers of  $k2, c, d$ .  $P1 = 4 \times 1$  matrix of the terms on L H S and  $P2 = 4 \times 1$  matrix of the terms on L H S.  $X1 = 4 \times 1$  matrix of values of  $K_1, a, b$  and  $X2 = 4 \times 1$  matrix of values of  $K_2, c$  and  $d$



Solving these equations using 'MATLAB' software we get values of unknown indices  $K_1$ ,  $K_2$ , a, b, c and d. Hence the model can be written in complete format by knowing values of indices calculated using Mat lab software.

## Conclusion

The mathematical models obtained from FDBM approach are in exponential form and complex as each independent major pie terms are multiple of various input parameters as described in Table 2. Interpretation of models can give extent of influence on output of the model by any pie terms or any variables listed in Table 2. Field data based modelling concept can found very useful and can be applied to any complex activity as the observations for variables are obtained directly from the work place and include all kind of data. Modelling and proper analysis can suggest a correct method of doing such activities and modifying the materials with changes in layout of workstation will improve production output, reduce losses of materials, losses due to error.

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